

8.2 Review Problems

I. Use Gaussian or Gauss Jordan elimination to solve.

$$x_1 - x_2 = 11$$

$$4x_1 + 3x_2 = -5$$

II. $2x_1 + 2x_2 = 0$

$$2x_1 + x_2 + x_3 = 0$$

$$3x_1 + x_3 = 0$$

8.3 Rank of a Matrix

Alvaro Rodriguez

05DEC2021

Find the rank of the given matrix.

$$3) \begin{bmatrix} 2 & 1 & 3 \\ 6 & 3 & 9 \\ -1 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

Find the rank of the given matrix.

$$7) \begin{bmatrix} 1 & -2 \\ 3 & -6 \\ 7 & -1 \\ 4 & 5 \end{bmatrix}$$

$$1. \begin{bmatrix} 0 & 2 & 4 & 0 \\ 1 & 2 & -2 & 3 \\ 5 & 1 & 0 & -1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Evaluate the indicated minor determinant or Cofactor

$$\boxed{8.4}$$

$$\boxed{P.8}$$

$$C_{23} = ?$$

$$M_{23} = ?$$

P. 24 | 8.4

Evaluate the determinant of the given matrix by Cofactor Expansion.

$$\begin{bmatrix} 1 & 1 & 1 \\ x & y & z \\ 2+x & 3+y & 4+z \end{bmatrix} = A$$

Section 8.5 - Questions

Brenton Kearney
Math 310-03

1) Evaluate the determinant of the matrix $B = \begin{pmatrix} 2a_1 & a_2 & a_3 \\ 6b_1 & 3b_2 & 3b_3 \\ 2c_1 & c_2 & c_3 \end{pmatrix}$

Using the result $A = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 5.$

2) Evaluate the determinant of the given matrix by reducing it to triangular form.

$$\begin{pmatrix} 2 & 4 & 5 \\ 4 & 2 & 0 \\ 8 & 7 & -2 \end{pmatrix}$$

8.6 Inverse of a Matrix

1. Find the inverse of the matrix

$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & 11 & 14 \\ -1 & 4 & 0 \end{bmatrix}$$

2. Use an inverse matrix to solve the system of equations

$$x_1 + x_3 = -4$$

$$x_1 + 6x_2 + x_3 = 0$$

$$5x_1 - x_2 = 6$$

8.6

#1

Cameron
Marshall

Verify that the matrix B is the inverse of matrix A

$$A = \begin{pmatrix} 1 & 1/2 \\ 2 & 3/2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

8.6

#7

Determine whether the given matrix is singular or nonsingular.
If it is nonsingular, find the inverse.

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 4 \\ 1 & -1 & 1 \end{pmatrix}$$

Cameron
Marshall

8.7: Cramer's Rule

MATH-310-03

Mia Tabladillo

10) In problems 1-10, solve the given system of equations by Cramer's Rule

$$4x + 3y + 2z = 8$$

$$-x + \quad 2z = 12$$

$$3x + 2y + z = 3$$

13) The magnitudes T_1 and T_2 of the tensions in the support wires shown in Figure 8.7.1 satisfy the equations

$$(\cos 25^\circ) T_1 - (\cos 15^\circ) T_2 = 0$$

$$(\sin 25^\circ) T_1 + (\sin 15^\circ) T_2 = 300$$

Use Cramer's rule to solve for T_1 and T_2 .

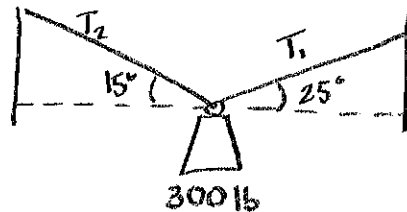


Figure 8.7.1 support wires in Problem 13

(H 8.8 #3)

Given: $A = \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix}$ $K_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $K_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $K_3 = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$

Determine if $K_1, K_2, + K_3$ are eigen vectors + if so what is their corresponding eigen value.

K_1

$$AK_1 = \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 18-6 \\ 6-2 \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$$

K_1 is not an eigen vector because AK_1 can't be written as λK_1 .

K_2 $AK_2 = \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6+0 \\ 2+0 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

K_2 is not an eigen vector because AK_2 can't be written as λK_2 .

K_3 $AK_3 = \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -5 \\ 10 \end{pmatrix} = \begin{pmatrix} -30+30 \\ -10+10 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0K_3$

$\lambda_3 = 0$ is an eigen value for K_3 which is an eigen vector of matrix A .

CH 8.8 #13

Given: $\begin{pmatrix} 4 & 8 \\ 0 & -5 \end{pmatrix}$

Find: eigen values & vectors
& If it is singular

$$\det(A - \lambda I) = 0$$

$$= \begin{vmatrix} 4-\lambda & 8 \\ 0 & -5-\lambda \end{vmatrix} = 0 \Rightarrow (4-\lambda)(-5-\lambda) - 0 = 0 \Rightarrow -(4-\lambda)(\lambda+5) = 0$$

$$\lambda_1 = 4, \lambda_2 = -5$$

Use $(A - \lambda I | 0)$ to find eigen vectors

$$\lambda_1 = 4$$

$$\left(\begin{array}{cc|c} 4-4 & 8 & 0 \\ 0 & -5-4 & 0 \end{array} \right) = \left(\begin{array}{cc|c} 0 & 8 & 0 \\ 0 & -9 & 0 \end{array} \right) \xrightarrow[\frac{1}{9}R_2]{\frac{1}{8}R_1} \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 1 & 0 \end{array} \right) \xrightarrow{R_2 - R_1} \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right)$$

$$\Rightarrow k_1 = 1, k_2 = 0$$

$$k_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ for } \lambda_1 = 4$$

Matrix A is non-singular
because it does not have the
eigen value $\lambda = 0$.

$$\lambda_2 = -5$$

$$\left(\begin{array}{cc|c} 4+5 & 8 & 0 \\ 0 & -5+5 & 0 \end{array} \right) = \left(\begin{array}{cc|c} 9 & 8 & 0 \\ 0 & 0 & 0 \end{array} \right) \Rightarrow 9k_1 + 8k_2 = 0$$
$$k_2 = -\frac{9}{8}k_1$$

$$k_2 = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \text{ for } \lambda_2 = -5$$

Assume $k_1 = -8$

$$k_2 = -\frac{9}{8}(-8) = 9$$

8.8 - Finding eigen values & characteristic polynomials

Daniel Rosales

1) Confirm that K is an eigenvector for A and find its eigenvalue.

$$A = \begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix}$$

$$K = \begin{bmatrix} 2 + \sqrt{2} \\ 2 \end{bmatrix}$$

2) Find the eigenvalues of A using the characteristic equation, state whether the matrix is singular or nonsingular. Then find the eigenvalues of the inverse matrix.

$$A = \begin{bmatrix} 1 & 6 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

2.2 Separable Equations

Howard S. Walker
7 December 2021

Solve the given Differential Equation by Separation of Variables.

#3

$$dx + e^{3x} dy = 0$$

#6

$$\frac{dy}{dx} + 2xy^2 = 0$$

Section 2.2

Ian Baker

8. Solve the given differential equation by separation of variables: $e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

24. Find an implicit and an explicit solution of the given initial-value problem:

$$\frac{dy}{dx} = \frac{y^2-1}{x^2-1}, \quad y(2) = 2$$

Conor O'Shaughnessy

Chapter 2.3 Review

- 1) Find the solution to the given equation.
Give the largest interval over which the general solution is found.

$$\frac{dy}{dx} + y = e^{3x}$$

- 2) Find the solution to the given equation.
Give the largest interval over which the general solution is defined

$$(x+2)^2 \frac{dy}{dx} = 5 - 8y - 4xy$$

$$(3x^2y + e^y) dx + (x^3 + xe^y - 2y) dy = 0$$

Juexin
maifan

exact

$$(2y \sin x \cos x - y + 2y^2 e^{xy^2}) dx = (x - \sin^2 x - 4xy e^{xy^2}) dy$$

Solution

	x	y	y'	Δy
$h=0.1$	1	5	-12	-1.2
	1.1	3.8	-8.2	-0.82
	1.2	2.98		
$h=0.05$	1	5	-12	-1.2
	1.05	4.4	-10.1	-0.505
	1.1	3.895	-8.665	-0.43325
	1.15	3.4707	-7.1122	-0.3556
	1.2	3.1151		

$$\Delta y = (h)(y')$$

$$y' = 2x - 3y + 1$$

$$y_2 = y_1 + (\Delta y)$$

$$= \triangleright \quad y' = (2)(1) - 3(5) + 1 = -12$$

$$\Delta y = (0.1)(-12)$$

$$y_2 = 5 - 1.2 = 3.8$$

Solution

$$y' = xy + \sqrt{y}$$

$$\text{for } y(0) = 1, h = 0.1$$

$$y' = (0)(1) + \sqrt{1} = 1$$

$$\Delta y = (0.1)(1) = 0.1$$

$$y_2 = 1 + 0.1 = 1.1$$

$$y_2 = y_1 + (\Delta y)$$

x	y	y'	Δy
0	1	1	0.1
0.1	1.1	1.159	0.1159
0.2	1.216	1.346	0.1346
0.3	1.350	1.567	0.1567
0.4	1.507	1.830	0.1830
0.5	1.690		

0	1	1	0.05
0.05	1.05	1.077	0.053
0.1	1.104	1.161	0.058
0.15	1.162	1.252	0.062
0.2	1.225	1.351	0.067
0.25	1.292	1.460	0.073
0.3	1.365	1.578	0.079
0.35	1.444	1.707	0.085
0.4	1.529	1.848	0.092
0.45	1.622	2.00	0.100
0.5	1.722		

$$h = 0.1$$

$$h = 0.05$$

2.4
making
exact

Juliana
Grigg

Making Equations Exact 2.4

$$30) (x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy = 0$$

$$\text{If: } \mu(x, y) = (x + y)^{-2}$$

$$34) \cos x dx + \left(1 + \frac{2}{y}\right) \sin x dy = 0$$

Kate Jordan - Section 1

2.7

Population / growth & decay rates

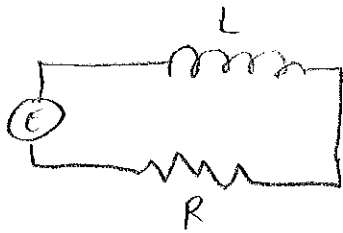
Problem #1: The population of a town grows at a rate proportional to the population present at time, t . The initial population of 500 increases by 15% in 10 years. What will the population be in 30 years? How fast is the population growing at $t = 30$?

Problem #2: A tank contains 200 liters of fluid in which 30 grams of salt is dissolved. Brine containing 1 gram of salt per liter is then pumped into the tank at a rate of 4 L/min. The well-mixed solution is also pumped out at the same rate. Find the number $A(t)$ of grams of salt in the tank at time, t .

2.7 #29

A 30-volt electromotive force is applied to an LR-series circuit in which the inductance is 0.1 henry and the resistance is 50 ohms. Find the current $i(t)$ if $i(0) = 0$. Determine the current as $t \rightarrow \infty$.

2.7. #29



Given:

$$L = 0.1 \text{ henry}$$

$$R = 50 \Omega$$

$$E = 30 \text{ V}$$

Find: $i(t)$ if $i(0) = 0$ current as $t \rightarrow \infty$

$$L \frac{di}{dt} + Ri = E(t)$$

$$\left(\frac{1}{10} \frac{di}{dt} + 50i = 30 \text{ V} \right) \text{ multiply by 10}$$

$$\frac{di}{dt} + 500i = 300 \text{ V}$$

$$P(t) = 500 \quad e^{500t} = \text{integrating factor}$$

$$\int P(t) dt = \int 500 dt = 500t$$

I.F.: e^{500t} Multiply Equation by I.F.,

$$e^{500t} \frac{di}{dt} + 500 e^{500t} i = 300 e^{500t}$$

$$\frac{d}{dt} \left[e^{500t} i \right] = 300 e^{500t}$$

$$e^{500t} i = \int 300 e^{500t} dt$$

$$e^{500t} i = \frac{3}{5} e^{500t} + C$$

$$i = \frac{3}{5} + C e^{-500t}$$

$$i(0) = 0 = \frac{3}{5} + C$$

$$C = -\frac{3}{5}$$

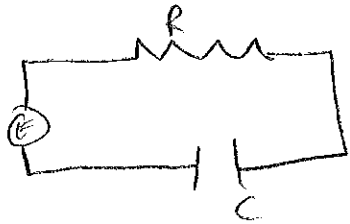
$$i(t) = \frac{3}{5} (1 - e^{-500t})$$

$$\lim_{t \rightarrow \infty} \frac{3}{5} (1 - e^{-500t}) = \boxed{\frac{3}{5} \text{ Amps}}$$

2.7 #32

A 200 volt electromotive force is applied to an RC-series circuit in which the resistance is 1000 ohms and the capacitance is 5×10^{-6} farad. Find the charge and current at $t = 0.005$ s. Determine the charge as $t \rightarrow \infty$.

2.7 #32



Given:

$$E = 200 \text{ V}$$

$$R = 1000 \ \Omega$$

$$C = 5 \times 10^{-6} \text{ F}$$

$$i(0) = 0.4$$

Find: $q(t)$

$$i \text{ @ } t = 0.0055$$

$$q \text{ as } t \rightarrow \infty$$

$$R \frac{dq}{dt} + \frac{1}{C} q = E(t)$$

$$(1000 \ \Omega \frac{dq}{dt} + \frac{1}{5 \times 10^{-6} \text{ F}} q = 200 \text{ V}) \text{ divide by } 1000$$

$$\frac{dq}{dt} + 200 q = \frac{1}{5}$$

$$P(t) = 200$$

$$\text{Integrating factor: } e^{\int P(t) dt} = e^{200t}$$

- multiply equation by integrating factor

$$e^{200t} \frac{dq}{dt} + 200 e^{200t} q = \frac{1}{5} e^{200t}$$

$$\frac{d}{dt} [e^{200t} q] = \frac{1}{5} e^{200t}$$

$$e^{200t} q = \frac{1}{1000} e^{200t} + C$$

$$q(t) = \frac{1}{1000} + C e^{-200t}$$

$$\frac{dq}{dt} = i(t) = -200 C e^{-200t}$$

$$i(0) = 0.4 = -200 C \Rightarrow i(t) = \frac{2}{5} e^{-200t}$$

$$C = -\frac{1}{500}$$

$$q(t) = \frac{1}{1000} - \frac{1}{500} e^{-200t}$$

$$i(0.0055) = \left[\frac{2}{5} e^{-1} \text{ A} \right]$$

$$\lim_{t \rightarrow \infty} \left(\frac{1}{1000} - \frac{1}{500} e^{-200t} \right) = \left[\frac{1}{1000} \text{ C} \right]$$

Homogeneous / Non-Homogeneous

3.1

Legend
Matantia

$$1) \quad 4y'' - 4y' - 3y = 0$$

$$2) \quad 4y'' - 12y' + 9y = 45x - 78$$

Maria Rossman 3.1 Problem #1.)

Find a member of the family
that is a solution of the
initial-value problem.

$$y = C_1 x + C_2 \ln x, \quad (0, \infty); \quad x^2 y'' - x y' + y = 0$$

• $y(1) = 3, \quad y'(1) = -1$

Maria Rossman 3.1 Problem #2.1

Determine whether the functions are linearly dependent or independent on $(-\infty, \infty)$.

$$f_1(x) = 5, \quad f_2(x) = \cos^2 x, \quad f_3(x) = \sin^2 x$$

Solve the initial-value problem:

$$4y'' - 4y' - 3y = 0 \quad y(0) = 1 \quad y'(0) = 5$$

Find the general solution of the higher-order differential equation:

$$y''' + 3y'' + 3y' + y = 0$$

①

$$y'' - y' - 12y = 0 \quad (-\infty, \infty)$$

$$y = e^{-3x} \quad y' = -3e^{-3x} \quad y'' = 9e^{-3x}$$

$$9e^{-3x} + (3e^{-3x}) - 12e^{-3x} = 0$$

$$0 = 0 \checkmark$$

$$y = e^{4x} \quad y' = 4e^{4x} \quad y'' = 16e^{4x}$$

$$16e^{4x} - 4e^{4x} - 12e^{4x} = 0$$

$$0 = 0 \checkmark$$

general

$$y = C_1 e^{-3x} + C_2 e^{4x}$$

$$\textcircled{2} \quad y'' - 2y' + 5y = 0; \quad e^x \cos 2x, \quad e^x \sin 2x \quad (-\infty, \infty)$$

$$y = e^x \cos 2x \quad y' = -2e^x \sin 2x + e^x \cos 2x \quad y'' = -3e^x \cos 2x - 4e^x \sin 2x$$

$$-3e^x \cos 2x - 4e^x \sin 2x + 4e^x \sin 2x - 2e^x \cos 2x + 5e^x \cos 2x = 0$$

$$0 = 0 \checkmark$$

$$y = e^x \sin 2x \quad y' = 2e^x \cos 2x + e^x \sin 2x \quad y'' = -3e^x \sin 2x + 4e^x \cos 2x$$

$$-3e^x \sin 2x + 4e^x \cos 2x - 4e^x \cos 2x + 2e^x \sin 2x + 5e^x \sin 2x = 0$$

$$0 = 0 \checkmark$$

general

$$y = C_1 e^x \sin 2x + C_2 e^x \cos 2x$$

3.3-complex roots (3.3 #11) [question] Maya Gonzales

#1 Find the general solution of the given second-order differential equation

$$y'' - 4y' + 5y = 0$$

3.3 - Complex roots (3.3 #39) [question] Maya Gonzalez

#2 solve the given boundary-value problem.

$$y'' + y = 0$$

$$y(0) = 0$$

$$y'(\pi/2) = 0$$

Section 3.6

Adam Bretsch

Math 310

12-7-21

14. Solve the given differential equation.

$$x^2 y'' - 7xy' + 4ly = 0$$

37. Find a homogenous Cauchy-Euler differential equation whose general solution is given.

$$y = c_1 \cos(\ln x) + c_2 \sin(\ln x)$$

3.6

Solve the given
differential equation

mikaela
morris

$$4x^2 y'' + y = 0$$

3.6

Solve the given differential equation
Mikaela Morris

$$\cancel{x^3} y''' - 6y = 0$$

Section 3.8: Spring-Mass, LRC-circuits

1. A 20-kilogram mass is attached to a spring. If the frequency of simple harmonic motion is $2/\pi$ cycles/s, what is the spring constant k ? What is the frequency of simple harmonic motion if the original mass is replaced with an 80-kilogram mass?

2. Find the charge on the capacitor in an LRC-series circuit at $t = 0.01$ s, when $L = 0.05$ h, $R = 2 \Omega$, $C = 0.01$ f, $E(t) = 0$ V, $q(0) = 5$ C, and $i(0) = 0$ A. Determine the first time at which the charge on the capacitor is equal to zero.

3.8 Mass/Spring & LRC-Circuits Problems

37 1. When a mass of 2 slug is attached to a spring whose constant is 32 N/m, it comes to rest in the equilibrium position. Starting at $t=0$, a force equal to $f(t) = 32e^{-2t} \cos 4t$ is applied to the system. Find the equation of motion in the absence of damping.

49 2. Find the charge on the capacitor in an LRC-series circuit at $t=0.01$ s when $L=0.05$ h, $R=2$ Ω , $C=0.01$ f, $E(t)=0$ V, $q(0)=5$ C and $i(0)=0$ A. Determine the first time at which the charge on the capacitor is equal to zero.

Section 4.3: Transforming Differential Equations into Algebraic Questions

$$\textcircled{1} \quad \mathcal{L}^{-1} \left\{ \frac{s/2 + s/3}{s^2 + 4s + 6} \right\}$$

$$\textcircled{2} \quad \text{Solve } y'' + 4y' + 6y = 1 + e^{-t}, \quad y(0) = 0, \quad y'(0) = 0.$$

Will Freking — 4.3 Translation on the t-axis

#48. ① $L^{-1}\left(\frac{e^{-2s}}{s^2(s-1)}\right) \rightarrow L^{-1}\left(\frac{1}{s^2(s-1)} e^{-2s}\right)$ * We know
 $L^{-1}(e^{-as} F(s)) = f(t-a) u(t-a)$
 $a=2$ $F(s) = \frac{1}{s^2(s-1)}$

② $L^{-1}(F(s)) = L^{-1}\left(\frac{1}{s^2(s-1)}\right)$

③ $\frac{1}{s^2(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} = \frac{As(s-1) + B(s-1) + Cs^2}{s^2(s-1)} \rightarrow$ ④ $1 = As(s-1) + B(s-1) + Cs^2$
 ⑤ $s=1$ $\frac{s=0}{1=C}$ $\frac{s=0}{B(-1)=1}$ $A+C=0 \rightarrow$ both $s^2!$
 $A=-C$
 $A=1$

⑥ so now we have

$\frac{1}{s^2(s-1)} = -\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s-1}$

⑦ $L^{-1}\left(\frac{1}{s^2(s-1)}\right) = L^{-1}\left(-\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s-1}\right)$
 $= L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{1}{s^2}\right) + L^{-1}\left(\frac{1}{s-1}\right)$

⑧ $L^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$ $n \neq 0$
 $L^{-1}\left(\frac{1}{s^2}\right) = \frac{1}{1!} L^{-1}\left(\frac{1!}{s^2}\right) = \frac{1}{1!} t = t$
 $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$ $a=1$
 $L^{-1}\left(\frac{1}{s-1}\right) = e^t$
 $L^{-1}\left(\frac{1}{s}\right) = \frac{1}{0!} t^0 = 1$

* Don't forget signs

⑨ $L^{-1}\left(\frac{1}{s^2(s-1)}\right) = -1 - t + e^t$

⑩ $L^{-1}\left(\frac{1}{s^2(s-1)} e^{-2s}\right) = -u(t-2) - (t-2)u(t-2) + e^{(t-2)}u(t-2)$

Inverse Laplace of $\frac{e^{-as}}{s}$ is always $u(t-a)$, with $s > 0!$

4.4

12

$$y'' + y = \sin t \quad y(0) = 1 \quad y'(0) = -1$$

$$24 \quad \mathcal{L}(t^2 * e^t)$$

Will Lawlor

4.9

Derivatives of
Transforms

Steven Jannicca

Section 2 - Math 310

4.4 Transforms of Integrals

19. $f(t) = 4t, g(t) = 3t^2$

p. 246

33. $\mathcal{L} \left\{ t \int_0^t \sin \tau d\tau \right\}$

p. 246